THE AFFINE-METRIC QUANTUM GRAVITY

WITH EXTRA LOCAL SYMMETRIES

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Abstract

We discuss the role of additional local symmetries related to the transformations of connection fields in the affine-metric theory of gravity. The corresponding BRST transformations connected with all symmetries (general coordinate, local Lorentz and extra) are constructed. It is shown, that extra symmetries give the additional contribution to effective action which is proportional to the corresponding Nielsen-Kallosh ghost one. Some arguments are given, that there is no anomaly associated with extra local symmetries.

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1 Introduction

At present, there is no theory of gravity that would be satisfactory ¿from the viewpoint of quantum field theory. The Einstein gravity, although agrees with all the available experimental data at the classical level, is not renormalizable. For the pure Einstein gravity, the one-loop counterterms vanish on shell [1] but the two-loop counterterms break the renormalizability of the model [2]. Furthermore, as one adds matter fields, the renormalizability is violated already at the one-loop level [3].

Therefore, one has either to modify the theory or to prove that the difficulties are due to imperfection of the methods applied to treat the model.

We are inclined to believe perturbative renormalizability to be one of the fundamental criteria for a "true" quantum gravity. We are going to assume that a perturbative renormalizable gravity exists but may differ from the Einstein theory.

The simplest way to modifying the Einstein gravity consists in adding some terms to the action which are quadratic in the curvature. This kind of a theory would be multiplicatively renormalizable and asymptotically free but not unitary [4]. Ghosts and tachyons would be present in the spectrum of the tree-level $S-$matrix, owing to the $\frac{1}{p^4}$ behavior of the metric field propagator, because the action involves higher derivatives. Attempts to restore unitarity by taking into account quantum corrections or adding some matter fields, failed [5].

We believe that the situation may be saved by increasing the symmetries of the initial action. Additional global or local symmetries that are maintained at the quantum level without generating anomalies may essentially improve renormalization properties. An example is supersymmetry. The simplest $N = 1$ supergravity is the first theory of gravity with matter in which the one-loop $S-$matrix is ultraviolet finite [6]. The known models of the $N = 1$ supergravity are finite up to two loops but may generate nonvanishing three-loop divergent counterterms. Models with the extended ($e.g. N = 8$) supersymmetry or some other additional symmetry (e.g. the local conformal symmetry) have better renormalization features [7], but there is no proof of their complete finiteness by now.

Another possibility is that space-time has a reacher structure than just a Riemann space. This implies introducing, besides the metric $g_{\mu\nu}$, some other geometric structures like torsion and nonmetricity.

The so-called Riemann-Cartan space is the simplest extension. It has the torsion as an independent dynamical variable. There is a wide literature on the gravity with torsion [8]. We only remind some facts concerning the renormalization properties. In the framework of the Riemann-Cartan space, one succeeded in constructing the models that are unitary at the tree level in the linear approximation [9]. Independent dynamical variables, metrics and torsion, have propagators with the $\frac{1}{p^2}$ behavior. However, at the one-loop level, the dimension-4 counterterms should be generated which are not forbidden by the symmetries present in the model, i.e. any scalars constructed from the contractions of the curvature and torsion tensors. In general, these counterterms will break either renormalizability or unitarity.

A further extension involves the affine-metric space-time characterized by the metric $g_{\mu\nu}$ and the affine connection $\bar{\Gamma}^{\sigma}{}_{\mu\nu}$ [10]. There are several variants proposed by now [11]. In the framework of the affine-metric theory there exist about two hundred arbitrary coefficients , which are not defined from basic principles. Because of the technical complication of this model, the renormalizability properties of the affine-metric quantum gravity have been studied insufficiently.

New hopes for a more perfect quantum gravity arose in connection with strings. The discussion of the bosonic string on the affine-metric manifold is given in [12].

Summarize the basic principles of the "true" quantum gravity:

- Basic assumption is the existence of a perturbatively renormalizable theory of gravity. As a consequence, all the assertions of the conventional perturbative quantum field theory should remain valid. In particular, the theory should be unitary at the tree level. Since no purely metric gravity is known to be both renormalizable and unitary, we are bound to introduce additional dynamical variables or to impose some new local symmetry, like supersymmetry. Thus, the true quantum perturbatively renormalizable, unitary theory of gravity without the supersymmetry can only be constructed in the framework of the extended space-time geometry, like the Riemann-Cartan or affine-metric geometry.

- The Lagrangian should not involve higher derivatives of the fields, in order to avoid ghosts and tachyons in the spectrum.

- The renormalization group analysis shows [13] that in an ordinary renormalizable quantum field theory the most essential role belongs to the terms with the dimension of the space-time. Consequently, it is natural to start constructing a renormalizable model from the terms with the corresponding dimension.

- For the classical limit, coinciding with the Einstein theory, to exist one needs to add a term linear in the curvature tensor to the Lagrangian.

In this paper, we consider the affine-metric gravity. This theory may be invariant with respect to some extra local transformations of the affine connection for special value of the coefficients [14]-[16]. These invariances restrict arbitrariness of the initial Lagrangian and avoid undesirable counterterms at the quantum level. However, the quantum properties of these symmetries have not been studied.

The main aim of this paper is to investigate extra local symmetries connected with the local transformation of the affine-metric gravity at the quantum level. We would like to show that these symmetries may give additional contribution to the effective action. The BRST-transformations connected with the symmetries of the affine-metric theory are constructed as a basis for further investigations of renormalizability and unitary properties. We don't discuss the physical ground and geometric nature of these symmetries in this paper. In section 2, we give the structure of the affine-metric gravity and introduce extra symmetries. In section 3, we introduce the BRST-transformations connected with extra symmetries in the geometrical and tetrad approaches, construct the quantum Lagrangian and discuss the problem of anomalies connected with new symmetries. In section 4, we conclude with a discussion of the results and perspective.

The following notation and conventions are accepted:

$$c = \hbar = 1; \qquad \mu, \nu = 0, 1, 2, 3; \qquad k^2 = 16\pi G, \qquad \varepsilon = \frac{4-d}{2}$$

$$(g) = det(g_{\mu\nu}), \qquad e = det(e^a{}_\mu) \qquad \eta_{\mu\nu} = (+ - - -),$$

The Riemannian connection is $\Gamma^\sigma{}_{\mu\nu} = g^{\sigma\lambda}\frac{1}{2}\left(-\partial_\lambda g_{\mu\nu} + \partial_\mu g_{\lambda\nu} + \partial_\nu g_{\lambda\mu}\right)$. Objects marked by bar are constructed by means of the affine connection $\bar{\Gamma}^\sigma{}_{\mu\nu}$. The others are the Riemannian objects. For further calculations one needs to define the following tensor object: $D^\sigma{}_{\mu\nu} = \bar{\Gamma}^\sigma{}_{\mu\nu} - \Gamma^\sigma{}_{\mu\nu}$.

2 Extra local symmetries in affine-metric gravity

The affine-metric manifold permits the geometric and tetrad description. The geometric approach implies the description in terms of the metric $g_{\mu\nu}$ and affine connection $\bar{\Gamma}^{\sigma}{}_{\mu\nu}$. The basic objects are expressed as

- curvature

$$\bar{R}^{\sigma}{}_{\lambda\mu\nu}(\bar{\Gamma}) = \partial_{\mu}\bar{\Gamma}^{\sigma}{}_{\lambda\nu} - \partial_{\nu}\bar{\Gamma}^{\sigma}{}_{\lambda\mu} + \bar{\Gamma}^{\sigma}{}_{\alpha\mu}\bar{\Gamma}^{\alpha}{}_{\lambda\nu} - \bar{\Gamma}^{\sigma}{}_{\alpha\nu}\bar{\Gamma}^{\alpha}{}_{\lambda\mu} \tag{1}$$

- torsion

$$\bar{Q}^{\sigma}{}_{\mu\nu}(\bar{\Gamma}) = \frac{1}{2}\left(\bar{\Gamma}^{\sigma}{}_{\mu\nu} - \bar{\Gamma}^{\sigma}{}_{\nu\mu}\right) \tag{2}$$

- nonmetricity

$$\bar{W}_{\sigma\mu\nu}(g,\bar{\Gamma}) = \bar{\nabla}_{\sigma}g_{\mu\nu} = \partial_{\sigma}g_{\mu\nu} - \bar{\Gamma}^{\alpha}{}_{\mu\sigma}g_{\alpha\nu} - \bar{\Gamma}^{\alpha}{}_{\nu\sigma}g_{\alpha\mu} \tag{3}$$

In the tetrad formalism, for describing the manifold we use the tetrad $e^{a}{}_{\mu}$ and the local Lorentz connection $\bar{\Omega}^{a}{}_{b\mu}$. Using the following relations [10]

$$g_{\mu\nu} = e^{a}{}_{\mu}e^{b}{}_{\nu}\eta_{ab} \tag{4}$$

$$\bar{\nabla}_{\sigma}e^{a}{}_{\mu} = \partial_{\sigma}e^{a}{}_{\mu} + \bar{\Omega}^{a}{}_{b\sigma}e^{b}{}_{\mu} - \bar{\Gamma}^{\nu}{}_{\mu\sigma}e^{a}{}_{\nu} = 0 \tag{5}$$

where η_{ab} is the Minkowskian metric, we can obtain the main geometric objects in tetrad formalism:

- curvature

$$\bar{R}^{\sigma}{}_{\lambda\mu\nu}(\bar{\Gamma}) = \bar{R}^{a}{}_{b\mu\nu}(\bar{\Omega})e_{a}{}^{\sigma}e^{b}{}_{\lambda} = (\partial_{\mu}\bar{\Omega}^{a}{}_{b\nu} - \partial_{\nu}\bar{\Omega}^{a}{}_{b\mu} + \bar{\Omega}^{a}{}_{\alpha\mu}\bar{\Omega}^{\alpha}{}_{b\nu} - \bar{\Omega}^{a}{}_{\alpha\nu}\bar{\Omega}^{\alpha}{}_{b\mu})e_{a}{}^{\sigma}e^{b}{}_{\lambda} \tag{6}$$

- torsion

$$\bar{Q}^{\sigma}{}_{\mu\nu}(\bar{\Gamma}) = \bar{Q}^{a}{}_{\mu\nu}(e,\bar{\Omega})e_{a}{}^{\sigma} = -\frac{1}{2}\left(\partial_{\mu}e^{a}{}_{\nu} - \partial_{\nu}e^{a}{}_{\mu} + \bar{\Omega}^{a}{}_{b\mu}e^{b}{}_{\nu} - \bar{\Omega}^{a}{}_{b\nu}e^{b}{}_{\mu}\right)e_{a}{}^{\sigma} \tag{7}$$

- nonmetricity

$$\bar{W}_{\sigma\mu\nu}(g,\bar{\Gamma}) = \bar{W}_{\sigma ab}(\bar{\Omega})e^{a}_{\mu}e^{b}_{\nu} = -\left(\bar{\Omega}_{ab\sigma} + \bar{\Omega}_{ba\sigma}\right)e^{a}_{\mu}e^{b}_{\nu} \tag{8}$$

An affine-metric theory of gravity may have additional local symmetries related to transformations of the connection [14]-[16]. The simplest is the transformation of irreducible parts of the connection. The affine connection can be rewritten as

$$\bar{\Gamma}^{\sigma}{}_{\mu\nu} = \Gamma^{\sigma}{}_{\mu\nu} + D^{\sigma}{}_{\mu\nu} \tag{9}$$

where $D^{\sigma}{}_{\mu\nu}$ is the tensor. An arbitrary tensor of third rank $D_{\sigma\mu\nu}$ is known to be expandable in terms of the following irreducible parts:

$$D_{\sigma\mu\nu} = A_{\sigma}g_{\mu\nu} + B_{\mu}g_{\nu\sigma} + C_{\nu}g_{\mu\sigma} + \frac{1}{6}\check{D}_{[\sigma\mu\nu]} + \underline{D}_{\sigma\mu\nu} \tag{10}$$

where $\check{D}_{[\sigma\mu\nu]}$ is the antisymmetric part; A_{σ}, B_{μ} and C_{ν}, are the vector fields

$$A_{\sigma} \equiv \frac{1}{18}\left(5D_{\sigma\lambda}{}^{\lambda} - D^{\lambda}{}_{\sigma\lambda} - D^{\lambda}{}_{\lambda\sigma}\right) \tag{11}$$

$$B_\sigma \equiv \frac{1}{18}\left(-D_{\sigma\lambda}{}^{\lambda} + 5D^{\lambda}{}_{\sigma\lambda} - D^{\lambda}{}_{\lambda\sigma}\right) \tag{12}$$

$$C_\sigma \equiv \frac{1}{18}\left(-D_{\sigma\lambda}{}^{\lambda} - D^{\lambda}{}_{\sigma\lambda} + 5D^{\lambda}{}_{\lambda\sigma}\right) \tag{13}$$

and $\underline{D}_{\sigma\mu\nu}$ is the traceless part satisfying the following conditions:

$$\underline{D}^{\nu}{}_{\mu\nu} = \underline{D}^{\nu}{}_{\nu\mu} = \underline{D}_{\nu\mu}{}^{\mu} \equiv 0 \tag{14}$$

$$\epsilon^{\lambda\sigma\mu\nu}\underline{D}_{\sigma\mu\nu} = 0 \tag{15}$$

The symmetries related to transformations of irreducible parts are

$$\bar{\Gamma}^{\sigma}{}_{\mu\nu} \to' \bar{\Gamma}^{\sigma}{}_{\mu\nu} = \bar{\Gamma}^{\sigma}{}_{\mu\nu} + g^{\sigma\rho}\Lambda_{[\rho\mu\nu]} + g^{\sigma\rho}\underline{T}_{\rho\mu\nu} + M^{\sigma}g_{\mu\nu} + N_\nu\delta^{\sigma}_{\mu} + P_\mu g^{\sigma}_{\nu} \tag{16}$$

where $\Lambda_{[\sigma\mu\nu]}, \underline{T}_{\sigma\mu\nu}, M_\sigma, N_\nu, P_\mu$ are arbitrary antisymmetric, traceless tensors and vectors, respectively. If the theory is invariant under all the symmetries (16), the affine connection has no dynamical degrees of freedom. Just becomes an auxiliary field. After eliminating it the theory reduced to a metric gravity. We restrict ourselves to considering only some particular cases of (16):

$$\bar{\Gamma}^{\sigma}{}_{\mu\nu} \to' \bar{\Gamma}^{\sigma}{}_{\mu\nu} = \bar{\Gamma}^{\sigma}{}_{\mu\nu} + g^{\sigma\lambda}\Lambda_{[\lambda\mu\nu]}(x) + \delta^{\sigma}_{\mu}C_\nu(x) \tag{17}$$

This transformation is the sum of the projective transformations [15]:

$$\bar{\Gamma}^{\sigma}{}_{\mu\nu} \to' \bar{\Gamma}^{\sigma}{}_{\mu\nu} = \bar{\Gamma}^{\sigma}{}_{\mu\nu} + \delta^{\sigma}_{\mu}C_\nu(x) \tag{18}$$

and antisymmetric transformations:

$$\bar{\Gamma}^{\sigma}{}_{\mu\nu} \to' \bar{\Gamma}^{\sigma}{}_{\mu\nu} = \bar{\Gamma}^{\sigma}{}_{\mu\nu} + g^{\sigma\lambda}\Lambda_{[\lambda\mu\nu]}(x) \tag{19}$$

Consider the tetrad approach. Using relations (5) it is easy to show that the transformation (17) has the following form in the tetrad formalism:

$$\bar{\Omega}^{a}{}_{b\sigma} \to' \bar{\Omega}^{a}{}_{b\sigma} = \bar{\Omega}^{a}{}_{b\sigma} + e^{a\lambda}\Lambda_{[\lambda b\sigma]}(x) + \delta^{a}_{b}C_\sigma(x) \tag{20}$$

What is the meaning of these extra gauge invariances? Usually, local invariances are related to physically relevant groups like, for instance, the diffeomorphism and Lorentz group or internal group in the Yang-Mills type theories. And usually, a gauge field (potential, connection) is adjoined to each gauge invariance. However, for extra local symmetries no additional gauge fields appear. Therefore, these symmetries do not fit into the framework of "ordinary" gauge theories. This is a gauge invariance without of any physical meaning and geometric nature. At the same time, the presence of extra gauge invariances imposes some new constraints on the source terms. These new constraints may solve some old problems concerning the interaction of matter and vector gauge fields with connection in affine-metric gravity [10], [11]. In our opinion, the role of these symmetries infers in excluding counterterms of the particular type and restricting arbitrariness of the initial Lagrangian. This is possible if the symmetries are maintained at the quantum level. The questions arise about the gauge fixing and corresponding ghost fields connected with these symmetries. For constructing the quantum Lagrangian we must add the gauge fixing terms and appropriate Faddeev-Popov

ghost fields. We derive the corresponding theory from the invariance of the full Lagrangian under the BRST-transformations

$$\mathbf{s}L_{quan} = 0 \tag{21}$$

where $\mathbf{s}$ is a graded, nilpotent BRST operator.

3 BRST-transformations in the affine-metric gravity with extra local invariances

Consider an arbitrary model of the affine-metric gravity quadratic in the torsion, curvature and nonmetricity, invariant under the general coordinate and additional local transformations (17) in the geometric approach. The propagators which stem from the classical Lagrangian are not all defined because the quadratic field approximation of the initial Lagrangian is degenerated, i.e. contains modes associated with the gauge invariance. The propagators can be made invertible if a gauge fixing term is added to initial Lagrangian. We consider the symmetries (17) as a gauge symmetry. Hence, we must fix these symmetries. The gauge fixing Lagrangian is

$$L_{gf} = \left(b^\mu \omega_{\mu\nu} F^\nu + \pi^\mu \zeta_{\mu\nu} f^\nu + d^\mu \varsigma_{\mu\nu} \rho^\nu - \frac{1}{2} b^\mu \omega_{\mu\nu} b^\nu - \frac{1}{2} \pi^\mu \zeta_{\mu\nu} \pi^\nu - \frac{1}{2} d^\mu \varsigma_{\mu\nu} d^\nu \right) \sqrt{-g} \tag{22}$$

where $\{b_\mu,\ \pi_\mu,\ d_\mu\}$ are auxiliary fields, $\{\omega_{\mu\nu}, \varsigma_{\mu\nu}, \zeta_{\mu\nu}\}$ are arbitrary differential operators which contain two or smaller derivatives and $\{F_\mu,\ f_\mu,\ \rho_\mu\}$ fix the general coordinate, projective and antisymmetric gauges.

Since auxiliary fields appear without derivatives in the Lagrangian, they can be eliminated by means of their equations of motion which yield

$$L_{gf} = \left(\frac{1}{2} F^\mu \omega_{\mu\nu} F^\nu + \frac{1}{2} f^\mu \zeta_{\mu\nu} f^\nu + \frac{1}{2} \rho^\mu \varsigma_{\mu\nu} \rho^\nu \right) \sqrt{-g} \tag{23}$$

We fix the general coordinate transformations by the following gauge:

$$F^\mu = (-g)^{-\tau} \partial_\nu \left(g^{\mu\nu} (-g)^\tau \right) + a_1 \bar{\Gamma}^\mu{}_{\alpha\beta} g^{\alpha\beta} + a_2 \bar{\Gamma}^\nu{}_{\alpha\nu} g^{\alpha\mu} + a_3 \bar{\Gamma}^\nu{}_{\nu\alpha} g^{\alpha\mu} \tag{24}$$

where τ and a_1, a_2, a_3 are arbitrary constants. The projective gauge (18) can be fixed as follows [16]:

$$f^\lambda = \left(f_1 \delta^\lambda_\sigma g^{\mu\nu} + f_2 g^{\mu\lambda} \delta^\nu_\sigma + f_3 g^{\nu\lambda} \delta^\mu_\sigma \right) \bar{\Gamma}^\sigma{}_{\mu\nu} \tag{25}$$

where $\{f_i\}$ are constants satisfying the condition:

$$f_1 + f_2 + 4 f_3 \neq 0 \tag{26}$$

The most general coordinate and projective gauge fixing terms are

$$F^\mu = (-g)^{-\tau} \partial_\nu \left(g^{\mu\nu} (-g)^\tau \right) + a_1 \bar{\Gamma}^\mu{}_{\alpha\beta} g^{\alpha\beta} + a_2 \bar{\Gamma}^\nu{}_{\alpha\nu} g^{\alpha\mu} + a_3 \bar{\Gamma}^\nu{}_{\nu\alpha} g^{\alpha\mu} + E^{\mu\rho\lambda}{}_\sigma{}^{\alpha\beta} \nabla_\rho \nabla_\lambda \bar{\Gamma}^\sigma{}_{\alpha\beta} \tag{27}$$

$$f^\lambda = \left(f_1 \delta^\lambda_\sigma g^{\mu\nu} + f_2 g^{\mu\lambda} \delta^\nu_\sigma + f_3 g^{\nu\lambda} \delta^\mu_\sigma \right) \bar{\Gamma}^\sigma{}_{\mu\nu} + G^{\lambda\rho\lambda}{}_\sigma{}^{\alpha\beta} \nabla_\rho \nabla_\lambda \bar{\Gamma}^\sigma{}_{\alpha\beta} \tag{28}$$

where $E^{\mu\rho\lambda}{}_{\sigma}{}^{\alpha\beta}$ and $G^{\mu\rho\lambda}{}_{\sigma}{}^{\alpha\beta}$ are reduction tensors, i.e. a product of metric tensors and Kroneker's symbols. In the initial Lagrangian the independent dynamical fields have propagators with the $\frac{1}{p^2}$ behavior. Then, the gauge-fixing terms proportional to $E^{\mu\rho\lambda}{}_{\sigma}{}^{\alpha\beta}$ and $G^{\mu\rho\lambda}{}_{\sigma}{}^{\alpha\beta}$ with higher derivatives may break the unitarity of the theory. To avoid this problem we consider the case $E^{\mu\rho\lambda}{}_{\sigma}{}^{\alpha\beta} = G^{\mu\rho\lambda}{}_{\sigma}{}^{\alpha\beta} = 0$.

For the antisymmetric transformation (19) we use the gauge condition

$$\rho^{\lambda} = \epsilon^{\lambda\sigma\mu\nu}\bar{\Gamma}_{\sigma\mu\nu} \tag{29}$$

The BRST-transformations are obtained in the usual way [17] ¿from gauge transformations by replacing the gauge parameter by the corresponding ghost field

$$\begin{array}{rclll} \mathbf{s}g_{\mu\nu} & = & \mathcal{L}_c g_{\mu\nu} & \mathbf{s}\bar{\Gamma}^{\sigma}{}_{\mu\nu} = \mathcal{L}_c\bar{\Gamma}^{\sigma}{}_{\mu\nu} + k\delta^{\sigma}_{\mu}\chi_{\nu} + k\eta^{\sigma}{}_{\mu\nu} & \\ \mathbf{s}\bar{c}_{\mu} & = & b_{\mu} & \mathbf{s}b_{\mu} = 0 & \mathbf{s}c^{\mu} = c^{\lambda}\partial_{\lambda}c^{\mu} \\ \mathbf{s}\bar{\chi}_{\mu} & = & \pi_{\mu} & \mathbf{s}\pi_{\mu} = 0 & \mathbf{s}\chi^{\nu} = 0 \\ \mathbf{s}\bar{\eta}_{\mu} & = & d_{\mu} & \mathbf{s}d_{\mu} = 0 & \mathbf{s}\eta^{\sigma}{}_{\mu\nu} = 0 \end{array} \tag{30}$$

where $\mathbf{s}$ is a graded, nilpotent BRST operator and $\{\bar{c}_{\nu}, c^{\mu},\}$, $\{\bar{\chi}_{\alpha}, \chi^{\beta}\}, \{\bar{\eta}_{\sigma}, \eta^{\lambda}{}_{\mu\nu}\}$ are anticommuting ghost fields connected with general coordinate, projective and antisymmetric transformations, respectively; $\mathcal{L}_{\xi}A^{\mu_1\ldots\mu_k}{}_{\nu_1\ldots\nu_l}$ is an ordinary Lie derivative. Under the general coordinate transformations $x^{\mu} \rightarrow' x^{\mu} = x^{\mu} + k\xi^{\mu}$,the Lie derivatives are:

$$\begin{array}{rcl} \mathcal{L}_{\xi}(-g)^{\alpha} & = & k\alpha\left(2\partial_{\sigma}\xi^{\sigma} + \xi^{\sigma}g^{\mu\nu}\partial_{\sigma}g_{\mu\nu}\right)(-g)^{\alpha} + O(k^2) \\ \mathcal{L}_{\xi}g_{\mu\nu} & = & k\left(\partial_{\mu}\xi^{\beta}g_{\beta\nu} + \partial_{\nu}\xi^{\beta}g_{\beta\mu} + \xi^{\beta}\partial_{\beta}g_{\mu\nu}\right) + O(k^2) \\ \mathcal{L}_{\xi}\bar{\Gamma}^{\sigma}{}_{\mu\nu}(x) & = & k\left(\bar{\Gamma}^{\sigma}{}_{\alpha\nu}\partial_{\mu}\xi^{\alpha} + \bar{\Gamma}^{\sigma}{}_{\mu\alpha}\partial_{\nu}\xi^{\alpha} - \bar{\Gamma}^{\alpha}{}_{\mu\nu}\partial_{\alpha}\xi^{\sigma} + \xi^{\alpha}\partial_{\alpha}\bar{\Gamma}^{\sigma}{}_{\mu\nu} + \partial_{\mu\nu}\xi^{\sigma}\right) \\ & + & O(k^2) \end{array} \tag{31}$$

The action of $\mathbf{s}$ on any function of fields is given by the graded Leibniz rule:

$$\begin{array}{rcl} \mathbf{s}\,(XY) & = & (\mathbf{s}X)\,Y \pm X\,(\mathbf{s}Y) \\ \mathbf{s}\partial_{\mu} & = & \partial_{\mu}\mathbf{s} \\ \mathbf{s}^2 & = & 0 \end{array} \tag{32}$$

where the minus sign occurs if X contains an odd number of ghosts and anti-ghosts.

The quantum Lagrangian is

$$L_{quant} = L_{clas} + \mathbf{s}\Big(\bar{c}^{\mu}\omega_{\mu\nu}\left(F^{\nu} - \frac{1}{2}b^{\nu}\right) + \bar{\chi}^{\mu}\zeta_{\mu\nu}\left(f^{\nu} - \frac{1}{2}\pi^{\nu}\right) + \bar{\eta}^{\mu}\varsigma_{\mu\nu}\left(\rho^{\nu} - \frac{1}{2}d^{\nu}\right)\Big) \tag{33}$$

where for simplicity we consider the case $\mathbf{s}\omega_{\mu\nu} = \mathbf{s}\zeta_{\mu\nu} = \mathbf{s}\varsigma_{\mu\nu} = 0$. The generating functional is

$$e^{iW} = \int dg_{\mu\nu}\; d\bar{\Gamma}^{\sigma}{}_{\mu\nu}\; d\bar{c}^{\mu}\; dc^{\nu}\; d\bar{\chi}^{\mu}\; d\chi^{\nu}\; d\bar{\eta}^{\mu}\; d\eta^{\sigma\lambda\nu} e^{iS_{quan}}\,(det\omega_{\mu\nu})^{\frac{1}{2}}\,(det\zeta_{\mu\nu})^{\frac{1}{2}}\,(det\varsigma_{\mu\nu})^{\frac{1}{2}} \tag{34}$$

where $det\omega_{\mu\nu}, det\zeta_{\mu\nu}, det\varsigma_{\mu\nu}$ are the so called Nielsen-Kallosh ghosts.

¿From (22) and (30) we obtain the one-loop ghost Lagrangian:

$$\begin{aligned} L_{gh} &= -\bar{c}^{\mu}\omega_{\mu\nu}\mathbf{s}F^{\nu} - \bar{\chi}^{\mu}\zeta_{\mu\nu}\mathbf{s}f^{\nu} - \bar{\eta}^{\mu}\varsigma_{\mu\nu}\mathbf{s}\rho^{\nu} \\ &= -\left(\bar{c}^{\mu}\ \bar{\chi}^{\mu}\ \bar{\eta}^{\mu}\right)\begin{pmatrix} \omega_{\mu\sigma}\triangle^{\sigma}{}_{\nu} & \omega_{\mu\nu}(a_1+a_2+4a_3) & 0 \\ \zeta_{\mu\sigma}Z^{\sigma}{}_{\nu} & (f_1+f_2+4f_3)\zeta_{\mu\nu} & 0 \\ \varsigma_{\mu\sigma}L^{\sigma}{}_{\nu} & 0 & \varsigma_{\mu}{}^{\alpha}\epsilon_{\alpha\sigma\lambda\nu} \end{pmatrix}\begin{pmatrix} c^{\nu} \\ \chi^{\nu} \\ \eta^{\sigma\lambda\nu} \end{pmatrix} \end{aligned} \quad (35)$$

where $\triangle^{\mu}{}_{\nu}, Z^{\mu}{}_{\nu}$ and $L^{\mu}{}_{\nu}$ are

$$\begin{aligned} \triangle_{\mu\nu} = \ & k\Big(\left(a_1-1\right)g_{\mu\nu}\nabla^2 + \frac{1}{2}\left(a_2+a_3+2\tau-1\right)\left(\nabla_{\mu}\nabla_{\nu}+\nabla_{\nu}\nabla_{\mu}\right) \\ & +\frac{1}{2}\left(2a_1-1-a_2-a_3-2\tau\right)R_{\mu\nu} + \left(a_1\nabla_{\nu}D_{\mu\lambda}{}^{\lambda} + a_2\nabla_{\nu}D^{\sigma}{}_{\mu\sigma} + a_3\nabla_{\nu}D^{\lambda}{}_{\lambda\mu}\right) \\ & + \left(a_2D^{\lambda}{}_{\nu\lambda} + a_3D^{\lambda}{}_{\lambda\nu}\right)\nabla_{\mu} - a_1\left(g_{\mu\nu}D^{\alpha\beta}{}_{\beta}\nabla_{\alpha} - D_{\mu\nu}{}^{\lambda}\nabla_{\lambda} - D_{\mu}{}^{\lambda}{}_{\nu}\nabla_{\lambda}\right)\Big) \\ & +O(k^2) \end{aligned} \quad (36)$$

$$\begin{aligned} Z_{\mu\nu} = \ & k\Big(f_1g_{\mu\nu}\nabla^2 + \frac{1}{2}\left(f_2+f_3\right)\left(\nabla_{\mu}\nabla_{\nu}+\nabla_{\nu}\nabla_{\mu}\right) + \frac{1}{2}\left(2f_1-f_2-f_3\right)R_{\mu\nu} \\ & + \left(f_1\nabla_{\nu}D_{\mu\lambda}{}^{\lambda} + f_2\nabla_{\nu}D^{\sigma}{}_{\mu\sigma} + f_3\nabla_{\nu}D^{\lambda}{}_{\lambda\mu}\right) + \left(f_2D^{\lambda}{}_{\nu\lambda} + f_3D^{\lambda}{}_{\lambda\nu}\right)\nabla_{\mu} \\ & - f_1\left(g_{\mu\nu}D^{\alpha\beta}{}_{\beta}\nabla_{\alpha} - D_{\mu\nu}{}^{\lambda}\nabla_{\lambda} - D_{\mu}{}^{\lambda}{}_{\nu}\nabla_{\lambda}\right)\Big) + O(k^2) \end{aligned} \quad (37)$$

$$L^{\mu}{}_{\nu} = k\epsilon^{\mu\alpha\beta\lambda}\left(\bar{Q}_{\alpha\beta\nu}\nabla_{\lambda} + D_{\alpha\nu\beta}\nabla_{\lambda} - g_{\alpha\nu}D^{\sigma}{}_{\beta\lambda}\nabla_{\sigma} + \nabla_{\nu}Q_{\alpha\beta\lambda}\right) + O(k^2) \quad (38)$$

Let us consider the case

$$a_1 + a_2 + 4a_3 = 0 \quad (39)$$

Then, we can get the diagonal form of the ghost Lagrangian (35) by the following redefinition of the ghost fields:

$$\begin{aligned} \tilde{\chi}^{\nu} &= \chi^{\nu} + \frac{1}{f_1+f_2+4f_3}Z^{\nu}{}_{\sigma}c^{\sigma} \\ \tilde{\eta}^{\sigma\mu\nu} &= \eta^{\sigma\mu\nu} + \frac{1}{6}\epsilon^{\sigma\mu\nu\lambda}Z_{\lambda\alpha}c^{\alpha} \end{aligned} \quad (40)$$

This redefinition does not change the functional integral measure. In the new variables the ghost Lagrangian has the diagonal form:

$$L_{gh} = -\left(\bar{c}^{\mu}\ \bar{\chi}^{\mu}\ \bar{\eta}^{\mu}\right)\begin{pmatrix} \omega_{\mu\sigma}\triangle^{\sigma}{}_{\nu} & 0 & 0 \\ 0 & (f_1+f_2+4f_3)\zeta_{\mu\nu} & 0 \\ 0 & 0 & \varsigma_{\mu}{}^{\alpha}\epsilon_{\alpha\sigma\lambda\nu} \end{pmatrix}\begin{pmatrix} c^{\nu} \\ \tilde{\chi}^{\nu} \\ \tilde{\eta}^{\sigma\lambda\nu} \end{pmatrix} \quad (41)$$

The loop contribution of the projective and antisymmetric ghosts to the effective action is proportional to $(det\zeta_{\mu\nu})\,(det\varsigma_{\alpha\beta})$. The one-loop generating functional is

$$e^{iW} = \int dg_{\mu\nu}\ d\bar{\Gamma}^{\sigma}{}_{\mu\nu} e^{i\left(S_{clas}+S_{gf}\right)} \left(det\omega_{\mu\nu}\right)^{\frac{1}{2}} \left(det\omega_{\mu\alpha}\triangle^{\alpha}{}_{\nu}\right) \left(det\zeta_{\mu\nu}\right)^{\frac{3}{2}} \left(det\varsigma_{\mu\nu}\right)^{\frac{3}{2}} \tag{42}$$

In this way, in the geometric formalism the projective and antisymmetric ghost contribution is added to the corresponding Nielsen-Kallosh ghost one. Hence, the presence of extra symmetries, which have not the physical meaning, give a new, extra contribution to the effective action. This contribution may improve the renormalizable properties of the theory.

In an analogous way consider the affine-metric gravity in the tetrad formalism. In the tetrad formalism the theory is invariant under the general coordinate, local Lorentz and additional (20) transformations. Let us construct the corresponding BRST-symmetry. The gauge fixing Lagrangian looks like

$$\begin{aligned} L_{gf} \quad = \quad & \Big(b^{\mu}\omega_{\mu\nu}F^{\nu} + \pi^{\mu}\zeta_{\mu\nu}f^{\nu} + d^{\mu}\varsigma_{\mu\nu}\rho^{\nu} + \lambda^{\mu\nu}\varrho_{\mu\nu\alpha\beta}f^{\alpha\beta} \\ & -\frac{1}{2}b^{\mu}\omega_{\mu\nu}b^{\nu} - \frac{1}{2}\pi^{\mu}\zeta_{\mu\nu}\pi^{\nu} - \frac{1}{2}d^{\mu}\varsigma_{\mu\nu}d^{\nu} - \frac{1}{2}\lambda^{\mu\nu}\varrho_{\mu\nu\alpha\beta}\lambda^{\alpha\beta}\Big)e \end{aligned} \tag{43}$$

where $\{b_{\mu},\ \pi_{\mu}, d_{\mu}, \lambda_{\mu\nu}\}$ are auxiliary fields, $\{\omega_{\mu\nu}, \varsigma_{\mu\nu}, \zeta_{\mu\nu}\varrho_{\mu\nu\alpha\beta}\}$ are arbitrary operators and $\{F_{\mu},\ f_{\mu\nu},\ f_{\mu},\ \rho_{\mu}\}$ fix the general coordinate, local Lorentz, projective and antisymmetric gauges. Since auxiliary fields appear without derivatives in the Lagrangian, they can be eliminated by means of their equations of motion which yield

$$L_{gf} = \Big(\frac{1}{2}F^{\mu}\omega_{\mu\nu}F^{\nu} + \frac{1}{2}f^{\mu}\zeta_{\mu\nu}f^{\nu} + \frac{1}{2}\rho^{\mu}\varsigma_{\mu\nu}\rho^{\nu} + \frac{1}{2}f^{\mu\nu}\varrho_{\mu\nu\alpha\beta}f^{\alpha\beta}\Big)e \tag{44}$$

We fix the coordinate, projective, antisymmetric and local Lorentz gauges by means of the following terms:

$$F^{\mu} = e^{-\tau}\partial_{\nu}\left(e_{a}{}^{\mu}e^{a\nu}e^{\tau}\right) + a_{1}\bar{\Omega}^{\mu\alpha}{}_{\alpha} + a_{2}\bar{\Omega}^{a\mu}{}_{a} + a_{3}\bar{\Omega}^{a}{}_{a}{}^{\mu} \tag{45}$$

$$f^{\lambda} = \left(f_{1}e_{a}^{\lambda}e^{b\nu} + f_{2}e^{b\lambda}e_{a}^{\nu} + f_{3}g^{\nu\lambda}\delta_{b}^{a}\right)\bar{\Omega}^{a}{}_{b\nu} \tag{46}$$

where τ and a_1, a_2, a_3 are an arbitrary constants and f_i are the constants satisfying the condition (26),

$$\rho^{\lambda} = \epsilon^{\lambda\mu\sigma\nu}\bar{\Omega}_{ab\nu}e^{a}_{\mu}e^{b}_{\sigma} \tag{47}$$

$$\begin{aligned} f_{ab} \quad = \quad & c_{1}\partial_{\mu}\left(\bar{\Omega}_{a}{}^{\mu}{}_{b} - \bar{\Omega}_{b}{}^{\mu}{}_{a}\right) + c_{2}\partial_{\mu}\left(\bar{\Omega}_{ab}{}^{\mu} - \bar{\Omega}_{ba}{}^{\mu}\right) \\ + \quad & c_{3}\partial_{\mu}\left(\bar{\Omega}^{\mu}{}_{ab} - \bar{\Omega}^{\mu}{}_{ba}\right) + c_{4}\left(e_{ab} - e_{ba}\right) \end{aligned} \tag{48}$$

where $\{_{i}\}$ are arbitrary constants.

The most general coordinate and projective gauge fixing terms are

$$F^{\mu} = e^{-\tau}\partial_{\nu}\left(e_{a}{}^{\mu}e^{a\nu}e^{\tau}\right) + a_{1}\bar{\Omega}^{\mu\alpha}{}_{\alpha} + a_{2}\bar{\Omega}^{a\mu}{}_{a} + a_{3}\bar{\Omega}^{a}{}_{a}{}^{\mu} + M^{\mu\alpha\beta}{}_{b}{}^{b\nu}\nabla_{\alpha}\nabla_{\beta}\bar{\Omega}^{a}{}_{b\nu} \tag{49}$$

$$f^{\lambda} = \left(f_{1}e_{a}^{\lambda}e^{b\nu} + f_{2}e^{b\lambda}e_{a}^{\nu} + f_{3}g^{\nu\lambda}\delta_{b}^{a}\right)\bar{\Omega}^{a}{}_{b\nu} + N^{\mu\alpha\beta}{}_{b}{}^{b\nu}\nabla_{\alpha}\nabla_{\beta}\bar{\Omega}^{a}{}_{b\nu} \tag{50}$$

where $M^{\mu\rho\lambda}{}_{\sigma}{}^{\alpha\beta}$ and $N^{\mu\rho\lambda}{}_{\sigma}{}^{\alpha\beta}$ are a product of metric tensors and Kroneker's symbols. To avoid the problems with unitarity of the model, we consider the case $M^{\mu\rho\lambda}{}_{\sigma}{}^{\alpha\beta} = N^{\mu\rho\lambda}{}_{\sigma}{}^{\alpha\beta} = 0$.

The complete BRST-transformations are

$$\begin{aligned}
\mathbf{s}e^{a}{}_{\mu} &= k\left(e^{a}{}_{\sigma}\partial_{\mu}c^{\sigma} + c^{\sigma}\partial_{\sigma}e^{a}{}_{\mu} + \Theta^{a}{}_{b}e^{b}{}_{\mu}\right) + O(k^2) \\
\mathbf{s}\bar{\Omega}^{a}{}_{b\mu} &= k\left(\bar{\Omega}^{a}{}_{b\sigma}\partial_{\mu}c^{\sigma} + c^{\sigma}\partial_{\sigma}\bar{\Omega}^{a}{}_{b\mu} + \bar{\nabla}_{\mu}\Theta^{a}{}_{b} + \delta^{a}_{b}\chi_{\mu} + \eta^{a}{}_{bm}\right) + O(k^2) \\
\mathbf{s}\bar{c}_{\mu} &= b_{\mu} \qquad \mathbf{s}b_{\mu} = 0 \qquad \mathbf{s}c^{\mu} = c^{\lambda}\partial_{\lambda}c^{\mu} \\
\mathbf{s}\bar{\chi}_{\mu} &= \pi_{\mu} \qquad \mathbf{s}\pi_{\mu} = 0 \qquad \mathbf{s}\chi^{\nu} = 0 \\
\mathbf{s}\bar{\eta}_{\mu} &= d_{\mu} \qquad \mathbf{s}d_{\mu} = 0 \qquad \mathbf{s}\eta^{\nu} = 0 \\
\mathbf{s}\bar{\Theta}_{a}{}^{b} &= \lambda_{a}{}^{b} \qquad \mathbf{s}\lambda_{a}{}^{b} = 0 \qquad \mathbf{s}\Theta^{a}{}_{b} = c^{\sigma}\partial_{\sigma}\Theta^{a}{}_{b} + \Theta^{a}{}_{m}\Theta^{m}{}_{b}
\end{aligned} \tag{51}$$

where $\mathbf{s}$ is a graded, nilpotent BRST operator and $\{\bar{c}_{\nu}, c^{\mu},\}$, $\{\bar{\chi}_{\alpha}, \chi^{\beta}\}$, $\{\bar{\eta}_{\sigma}, \eta^{\lambda}{}_{\mu\nu}\}$, $\{\bar{\Theta}_{m}^{\ n}, \Theta^{a}{}_{b}\}$ are anticommuting ghost fields connected with the general coordinate, projective, antisymmetric and the local Lorentz transformations, respectively.

The quantum Lagrangian is

$$\begin{aligned}
L_{quant} = L_{clas} + \mathbf{s}\Bigg(&\bar{c}^{\mu}\omega_{\mu\nu}\left(F^{\nu} - \frac{1}{2}b^{\nu}\right) + \bar{\chi}^{\mu}\zeta_{\mu\nu}\left(f^{\nu} - \frac{1}{2}\pi^{\nu}\right) \\
&+\bar{\eta}^{\mu}\varsigma_{\mu\nu}\left(\rho^{\nu} - \frac{1}{2}d^{\nu}\right) + \bar{\Theta}^{\alpha\beta}\varrho_{\alpha\beta\mu\nu}\left(f^{\mu\nu} - \frac{1}{2}\lambda^{\mu\nu}\right)\Bigg)
\end{aligned} \tag{52}$$

where for simplicity we consider the case $\mathbf{s}\omega_{\mu\nu} = \mathbf{s}\zeta_{\mu\nu} = \mathbf{s}\varsigma_{\mu\nu} = \mathbf{s}\varrho_{\alpha\beta\mu\nu} = 0$. The generating functional has the following form

$$\begin{aligned}
e^{iW} &= \int de^{a}{}_{\mu}\, d\bar{\Omega}^{a}{}_{b\nu}\, d\bar{\Theta}^{ab}\, d\Theta^{mn}\, d\bar{c}^{\mu}\, dc^{\nu}\, d\bar{\chi}^{\mu}\, d\chi^{\nu}\, d\bar{\eta}^{\mu}\, d\eta^{\sigma\lambda\nu} e^{iS_{quan}} \\
&\quad (det\omega_{\mu\nu})^{\frac{1}{2}}\, (det\zeta_{\mu\nu})^{\frac{1}{2}}\, (det\varsigma_{\mu\nu})^{\frac{1}{2}}\, (det\varrho_{abmn})^{\frac{1}{2}}
\end{aligned} \tag{53}$$

The ghost Lagrangian is

$$\begin{aligned}
L_{gh} &= -\bar{c}^{\mu}\omega_{\mu\nu}\mathbf{s}F^{\nu} - \bar{\chi}^{\mu}\zeta_{\mu\nu}\mathbf{s}f^{\nu} - \bar{\eta}^{\mu}\varsigma_{\mu\nu}\mathbf{s}\rho^{\nu} - \bar{\Theta}^{\mu\nu}\varrho_{\mu\nu\alpha\beta}\mathbf{s}f^{\alpha\beta} = \\
&- \left(\bar{c}^{\mu}\ \bar{\chi}^{\mu}\ \bar{\eta}^{\mu}\ \bar{\Theta}^{ab}\right)\begin{pmatrix} \omega_{\mu\alpha}\triangle^{\alpha}_{\ \nu} & B\omega_{\mu\nu} & 0 & K\omega_{\mu m}\bar{\nabla}_{n} \\ \zeta_{\mu\alpha}Z^{\alpha}_{\ \nu} & A\zeta_{\mu\nu} & 0 & L\zeta_{\mu m}\bar{\nabla}_{n} \\ \varsigma_{\mu\alpha}D^{\alpha}_{\ \nu} & 0 & \varsigma_{\mu\alpha}\epsilon^{\alpha\sigma\lambda\nu} & \varsigma_{\mu\alpha}\epsilon^{\alpha\beta mn}\bar{\nabla}_{\beta} \\ \varrho_{abij}M^{ij}{}_{\nu} & U\varrho_{ab\mu\nu}\partial_{\mu} & T\varrho_{ab\lambda\nu}\partial_{\sigma} & \varrho_{abij}S^{ij}{}_{mn} \end{pmatrix}\begin{pmatrix} c^{\nu} \\ \chi^{\nu} \\ \eta_{\sigma\lambda\nu} \\ \Theta^{mn} \end{pmatrix}
\end{aligned} \tag{54}$$

where $A = f_1 + f_2 + 4f_3$, $B = a_1 + a_2 + 4a_3$, $K = (a_1 - a_2)$, $L = (f_1 - f_2)$, $T = 2(c_2 + c_3 - c_1)$, $U = 2(c_1 + c_3)$, and $\triangle^{\mu}{}_{\nu}, D^{\mu}{}_{\nu}, Z^{\mu}{}_{\nu}, M^{a}{}_{b\nu}$ and $S^{ab}{}_{mn}$ are

$$\begin{aligned}
\triangle_{\mu\nu} &= k\Bigg(-g_{\mu\nu}\nabla^2 + \frac{1}{2}(\tau - 1)(\nabla_{\mu}\nabla_{\nu} + \nabla_{\nu}\nabla_{\mu}) - \frac{1}{2}(1+\tau)R_{\mu\nu} \\
&\quad +a_1\left(\bar{\Omega}_{\mu}{}^{\sigma}{}_{\nu}\partial_{\sigma} + \partial_{\nu}\bar{\Omega}_{\mu\sigma}{}^{\sigma}\right) + a_2\left(\bar{\Omega}^{\sigma}{}_{\mu\nu}\partial_{\sigma} + \partial_{\nu}\bar{\Omega}^{\sigma}{}_{\mu\sigma}\right) \\
&\quad +a_3\left(\bar{\Omega}^{\sigma}{}_{\sigma\nu}\partial_{\mu} + \partial_{\nu}\bar{\Omega}^{\sigma}{}_{\sigma\mu}\right)\Bigg) + O(k^2)
\end{aligned} \tag{55}$$

$$Z_{\mu\nu} = k\Big(f_1\left(\bar{\Omega}_{\mu\ \nu}^{\ \sigma}\partial_\sigma + \partial_\nu\bar{\Omega}_{\mu\sigma}^{\ \ \sigma}\right) + f_2\left(\bar{\Omega}^\sigma_{\ \mu\nu}\partial_\sigma + \partial_\nu\bar{\Omega}^\sigma_{\ \mu\sigma}\right)$$
$$+f_3\left(\bar{\Omega}^\sigma_{\ \sigma\nu}\partial_\mu + \partial_\nu\bar{\Omega}^\sigma_{\ \sigma\mu}\right)\Big) + O(k^2) \qquad (56)$$

$$D^\mu_{\ \nu} = k\epsilon^{\mu\alpha\beta\lambda}\left(\bar{\Omega}_{\alpha\beta\nu}\partial_\lambda + \partial_\nu\bar{\Omega}_{\alpha\beta\lambda}\right) + O(k^2) \qquad (57)$$

$$S_{abmn} = (\eta_{am}\eta_{bn} - \eta_{an}\eta_{bm})\left(c_2\partial_\mu\bar{\nabla}_\mu + c_4\right)$$
$$+ \frac{(c_1-c_3)}{2}\left(\partial_n\bar{\nabla}_b\eta_{ma} - \partial_m\bar{\nabla}_b\eta_{na} - \partial_n\bar{\nabla}_a\eta_{mb} + \partial_m\bar{\nabla}_a\eta_{bn}\right) \qquad (58)$$

$$M_{ab\nu} = c_1\left(\partial_\mu\left(\bar{\Omega}_{a\ \nu}^{\ \mu}\partial_b - \bar{\Omega}_{b\ \nu}^{\ \mu}\partial_a + \partial_\nu\bar{\Omega}_{a\ b}^{\ \mu} - \partial_\nu\bar{\Omega}_{b\ a}^{\ \mu}\right)\right)$$
$$+ c_2\left(\partial_\mu\left(\bar{\Omega}_{ab\nu}\partial_\mu - \bar{\Omega}_{ba\nu}\partial_\mu + \partial_\nu\bar{\Omega}_{ab}^{\ \ \mu} - \partial_\nu\bar{\Omega}_{ba}^{\ \ \mu}\right)\right)$$
$$+ c_4\left(\eta_{a\nu}\partial_b - \eta_{\nu a}\partial_a + \partial_\nu e_{ab} - \partial_\nu e_{ba}\right) + O(k^2) \qquad (59)$$

Let us consider the case $B = K = L = T = U = 0$. Define new variables

$$\tilde{\chi}^\nu = \chi^\nu + \frac{1}{A}Z^\nu_{\ \sigma}c^\sigma$$
$$\tilde{\eta}^{\sigma\lambda\nu} = \eta^{\sigma\lambda\nu} + \frac{1}{6}\epsilon^{\sigma\lambda\nu\mu}Z_{\mu\alpha}c^\alpha$$
$$\tilde{\Theta}^a_{\ b} = \Theta^a_{\ b} + M^a_{\ b\sigma}c^\sigma \qquad (60)$$

This redefinition does not change the functional integral measure. In the new variables the ghost Lagrangian has the diagonal form:

$$L_{gh} = \left(\bar{c}^\mu\ \bar{\chi}^\mu\ \bar{\eta}^\mu\ \bar{\Theta}^{ab}\right)\begin{pmatrix} \omega_{\mu\alpha}\triangle^\alpha_{\ \nu} & 0 & 0 & 0 \\ 0 & A\zeta_{\mu\nu} & 0 & 0 \\ 0 & 0 & \varsigma_{\mu\alpha}\epsilon^{\alpha\sigma\lambda\nu} & 0 \\ 0 & 0 & 0 & \varrho_{abij}S^{ij}_{\ \ mn} \end{pmatrix}\begin{pmatrix} c^\nu \\ \tilde{\chi}^\nu \\ \tilde{\eta}_{\sigma\lambda\nu} \\ \tilde{\Theta}^{mn} \end{pmatrix} \qquad (61)$$

We see that in the tetrad formalism the projective and antisymmetric ghosts also give the contribution in the effective action. The one-loop generating functional is

$$e^{iW} = \int de^a_{\ \mu}\, d\bar{\Omega}^a_{\ b\nu}\, d\bar{\Theta}^{ab}\, d\Theta^{mn}\, d\bar{c}^\mu\, dc^\nu\, d\bar{\chi}^\mu\, d\chi^\nu\, d\bar{\eta}^\mu\, d\eta^{\sigma\lambda\nu} e^{i\left(S_{clas}+S_{gf}\right)}$$
$$(det\omega_{\mu\nu})^{\frac{1}{2}}(det\zeta_{\mu\nu})^{\frac{3}{2}}(det\varsigma_{\mu\nu})^{\frac{3}{2}}(det\varrho_{abmn})^{\frac{1}{2}}(det\omega_{\mu\alpha}\triangle^\alpha_{\ \nu})\left(det\varrho_{abij}S^{ij}_{\ \ mn}\right) \qquad (62)$$

This contribution is added to the corresponding Nielsen-Kallosh ghost one.

Consider the question of anomalies, related to transformations (16). Anomaly is violation of some classical symmetries at the quantum level. Anomalies (like the well known Adler-Bell-Jackiw anomaly) may arise in the case when the variation of the action explicitly depends on the space-time dimension. If there is no such dependence, the dimensional regularization

retain the symmetry of the regularized model. The action principles [19], guarantees that the minimal subtraction scheme [20] will not break the symmetry after renormalization.

Transformations (16) and the variation of the affine-metric action do not explicitly depend on the space-time dimension. Consequently, there is no anomaly in the theory associated with these transformations.

4 Conclusion

A lot of unsolved problems in quantum gravity make one to search for new ways to solve them. At present, no model is quite satisfactory from the viewpoint of quantum field theory, possessing unitarity, renormalizability, existing S-matrix, etc. A criterion of the physical significance of the results of loop calculation can be the gauge and parametric independence [21].

In the present paper, we have considered the affine-metric gravity with extra local symmetries (16) related to the transformations of affine connection. These symmetries do not have the "ordinary" physical meaning and geometrical nature: for the extra local symmetries no additional gauge fields appear. The role of these symmetries is to suppress the counterterms that break the renormalizability of the model and restrict arbitrariness of the initial Lagrangian. Besides, these new symmetries imposes new constraints on the source term. Although they are local symmetries we have shown that no corresponding anomalies are generated. We have constructed the BRST-transformations connected with extra symmetries in geometric (30) and tetrad (51) approaches and shown that in both formalisms these symmetries give the additional contribution to the effective action (see (42) and (62), respectively) which is proportional to the corresponding Nielsen-Kallosh ghost one. These additional contributions may improve the renormalization properties of the theory.

One of the unresolved problems is to find the full set of extra symmetries, existing in affine-metric gravity. We know only two ways to solve this problem. The first one is to postulate that these extra symmetries have some physical sense [2]. The other way is to finds the full set of first-class constraints using the hamiltonian formalism [14]. But this method does not allow us to understand the physical ground of extra local symmetries.

We don't know the full set of extra local symmetries in affine-metric gravity. In present paper we discussed only two kinds of these symmetries : the projective (18) and antisymmetric (19) ones. There exist two interesting particular cases of the considered extra symmetries. In these cases the parameter of extra transformation is the derivative of some field [9], [14]:

$$\bar{\Gamma}^{\sigma}{}_{\mu\nu} \to' \bar{\Gamma}^{\sigma}{}_{\mu\nu} = \bar{\Gamma}^{\sigma}{}_{\mu\nu} + \delta^{\sigma}_{\mu}\partial_{\nu}\Phi(x) + g^{\sigma\alpha}\partial_{[\alpha}\lambda_{\mu\nu]}(x) \tag{63}$$

The general result of this paper does not change for these cases: it is necessary to fix the extra local symmetries and then these symmetries will give the non-trivial contribution to the one-loop divergences. To proof this statement one needs only to modify a little the gauge conditions (25) and (29).

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[2]In fact, these extra symmetries give rise to some deformation of the initial connection. This deformed connection can be used for construction of the Grand Unified Theory in the framework of the supergravity.

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